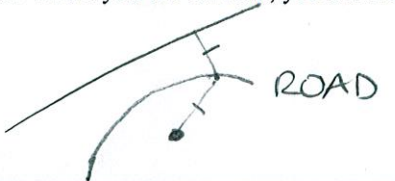
Write the definition of a hyperbola. **Use complete sentences and proper English as shown in lecture.** **SCORE: ___ / 4 PTS**

A HYPERBOLA IS THE LOCUS OF POINTS IN THE PLANE WHOSE DISTANCES TO TWO FIXED POINTS (FOCI) DIFFER BY A FIXED CONSTANT

Chris's house is 2 miles from Hunter Street (which is a straight road). There is a road in Chris's town such that, **SCORE: ___ / 2 PTS**
no matter where you are on road, you are the same distance from Chris's house as you are from Hunter Street. What is the shape of that road ?



PARABOLA (2)

Using the definition of a hyperbola, find the equation of the hyperbola such that the distances from any point on the hyperbola to the foci $(0, \pm 6)$ differ by 6.

SCORE: ___ / 8 PTS

IF (x, y) IS ON THE HYPERBOLA

(1) $\sqrt{x^2+(y+6)^2} - \sqrt{x^2+(y-6)^2} = \pm 6$

(1) $\sqrt{x^2+(y+6)^2} = \sqrt{x^2+(y-6)^2} \pm 6$

(1) $x^2+y^2+12y+36 = x^2+y^2-12y+36 \pm 12\sqrt{x^2+(y-6)^2}+36$

(1) $24y - 36 = \pm 12\sqrt{x^2+(y-6)^2}$

(1) $2y - 3 = \pm\sqrt{x^2+(y-6)^2}$

(1) $4y^2-12y+9 = x^2+y^2-12y+36$

(1) $3y^2-x^2 = 27$

(1) $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Find the standard form of the equation of the ellipse with foci $(-5, 3)$ and $(-5, -1)$ and a minor axis of length 8.

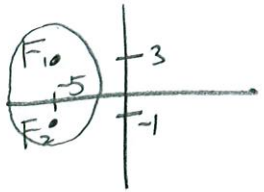
SCORE: ___ / 6 PTS

CENTER $= \left(-5, \frac{3+(-1)}{2}\right) = (-5, 1)$ (1/2)

$c = 3 - 1 = 2$ (1/2)

$2b = 8 \rightarrow b = 4$ (1)

$a^2 = 4^2 + 2^2 = 20$ (1)



$$\frac{(x+5)^2}{16} + \frac{(y-1)^2}{20} = 1$$

(1) (1/2) (1) (1/2)

Find the focus and directrix of the parabola with equation $y = -\frac{1}{2}x^2 - x + \frac{1}{2}$.

SCORE: ___ / 7 PTS

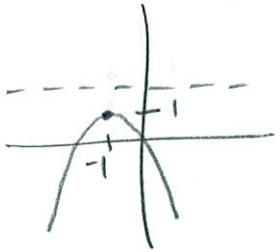
$-2y = x^2 + 2x - 1$

$-2y + 1 = x^2 + 2x$

$-2y + 2 = x^2 + 2x + 1$ (2)

$(x+1)^2 = -2(y-1)$ (1)

$4p = -2 \rightarrow p = -\frac{1}{2}$ (1)



VERTEX $= (-1, 1)$

FOCUS $= (-1, 1 - \frac{1}{2})$

$= (-1, \frac{1}{2})$ (1)

DIRECTRIX

$y = 1 + \frac{1}{2}$

(1) $y = \frac{3}{2}$ (1)

Consider the ellipse with equation $4x^2 + y^2 - 8x + 12y + 24 = 0$.

SCORE: ___ / 8 PTS

[a] Find the standard form of the equation of the ellipse.

$4x^2 - 8x + y^2 + 12y = -24$ (1, ½)

$4(x^2 - 2x + 1) + (y^2 + 12y + 36) = -24 + 4 \cdot 1 + 36$ (1)

$4(x-1)^2 + (y+6)^2 = 16$ (1)

$\frac{(x-1)^2}{4} + \frac{(y+6)^2}{16} = 1$ (1½)

[b] Find the foci of the ellipse.

CENTER $= (1, -6)$

$16 = 4 + c^2$

$c = \sqrt{12} = 2\sqrt{3}$ (1)

FOCI $= (1, -6 \pm 2\sqrt{3})$ (2)